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SPHERICALLY-SYMMETRIC SHOCK WAVE IN A DILATANT MEDIUM*

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The problem is examined of the expansion of a spherical region filled with a gas with high initial pressure, in a continuous medium with dilatancy, by which is meant the kinematic connection between the velocities of the dilatational and the maximal tangential deformations. The change in the medium's density behind the shock wave's front takes place only due to this kinematic connection. The model given was examined in a number of papers (/1−3/ and others) mainly for describing the behavior of certain soils in an underground explosion. Previously the soil density behind the shock wave's front had been reckoned to be constant /4−6/. An analogous problem is examined below in the case when the dilatancy velocity coefficient is a piecewise-constant function of the medium's density.

We consider an expanding spherical cavity (a cavern) having an initial radius a_0 and an initial pressure p_{k0}. Going ahead of the cavity's expansion, a shock wave is propagated over the medium, behind whose front we assume the fulfillment of Coulomb's law

$$(\sigma_r-\sigma_\varphi)/2=-k+m(\sigma_r+\sigma_\varphi)/2 \tag{1}$$

where k and m are known constants, σ_r and σ_φ are the stresses in the radial and orthogonal to its directions, respectively. The medium between the cavity and the shock wave's front is described by the momentum- and mass-preserving equations and the dilatancy equation

$$\begin{aligned}&\rho(\partial u/\partial t+u\partial u/\partial r)=\partial r\sigma_r/\partial r+2(\sigma_r-\sigma_\varphi)/r\\&\partial\rho/\partial t+u\partial\rho/\partial r+\rho(\partial u/\partial r+2u/r)=0\\&\partial u/\partial r+2u/r=\Lambda(\rho)(u/r-\partial u/\partial r)\end{aligned} \tag{2}$$

Here ρ is the medium's density, u is the mass velocity, r is the radius, t is time and $\Lambda(\rho)$ is the dilatancy velocity. At the wavefront we begin with the mass- and momentum-preservation laws

$$u_*(t)=\varepsilon_1(t)R^{\cdot}(t),\quad p_*(t)-p_0=\rho_0\varepsilon_1(t)R^{\cdot 2}(t) \tag{3}$$

Here $R(t)$ and $R^{\cdot}(t)$ are the radius and the velocity of the shock wave's front, $p_0=\rho_0gh$ is the lithostatic pressure at depth h, $\varepsilon_1=1-\rho_0/p_*$ is the compression shock at the front, $p_*(t)=-\sigma_{r*}(t)$ is the pressure at the front. The asterisk denotes the values of quantities at the shock wave's front, while the subscript zero denotes the values in the unperturbed medium.

We neglect the density's dependence on temperature; therefore, the system of Eqs.(2) and (3) can be solved without bringing in the energy balance equation. It is convenient to write Eqs.(2) in the Lagrangian coordinates (r_0, t). Denoting the new functions of the variables (r_0, t) by the same letters u, ρ, σ_r, r, with due regard to (1) we obtain the fulfillment of the equations

$$\begin{aligned}&\rho_0r_0^2r^{\alpha-2}\frac{\partial u}{\partial t}+\frac{\partial}{\partial r_0}\left[r^\alpha\left(p+\frac{k}{3m}\right)\right]=0\\&\frac{\partial r}{\partial r_0}=\frac{\rho_0r_0^2}{\rho r^2},\quad \Lambda(\rho)\frac{\partial}{\partial t}\ln(\rho r^3)+\frac{\partial}{\partial t}\ln\rho=0\\&(\alpha=4m/(1+m),\ p(r_0,t)=-\sigma_r(r_0,t))\end{aligned} \tag{4}$$

in the region behind the front. We take it that the rock being destroyed at the front reaches its own limit compression ($\rho_*(t)=\mathrm{const}$), while the dilatancy velocity $\Lambda(\rho)$ is approximated by a piecewise-constant function

$$\Lambda=\Lambda_1,\ \rho_1<\rho<\rho_*;\ \Lambda=\Lambda_2,\ \rho_2<\rho<\rho_1;\ \Lambda=0,\ \rho<\rho_2 \tag{5}$$

where $\Lambda_1, \Lambda_2, \rho_1, \rho_2$ are constants. The value $\Lambda=0$ corresponds to the case of an incompressible

*Prikl.Matem.Mekhan.,46,No.2,pp.349-352,1982 medium, which is usually postulated in papers on soils (/4-6/ and others), while when $\Lambda > 0$ the last equation in (2) or the last equation in (4), equivalent to it in the Lagrangian coordinates, describes the mellowing of the medium. The assumptions made permit us to pass from the system of partial differential Eqs.(4) to an ordinary differential equation in the front's radius $R(t)$ (the camouflet equation).

We denote by $a(t)$ the cavity's radius, by t_1 and $r_1(t)$ the time and the radius for which the density ρ_1 first is achieved, by t_2 and $r_2(t)$ the time and the radius for which the density ρ_2 first is achieved. The corresponding Lagrangian coordinates are $a_0 = \mathrm{const}$, $r_{01}(t)$, $r_{02}(t)$. The whole medium is separated into domains: a domain D_0 in which $\rho = \rho_0$, $r > R(t)$; a domain D_1 in which $\rho_1 < \rho < \rho_*$, $a(t) < r < R(t)$ when $t < t_1$ and $r_1(t) < r < R(t)$ when $t > t_1$; a domain D_2 in which $\rho_2 < \rho < \rho_1$, $a(t) < r < r_1(t)$ when $t_1 < t < t_2$ and $r_2(t) < r < r_1(t)$ when $t > t_2$; a domain D_3 in which $\rho = \rho_2$, $a(t) < r < r_2(t)$ when $t > t_2$. This separation is shown schematically in Fig.1.

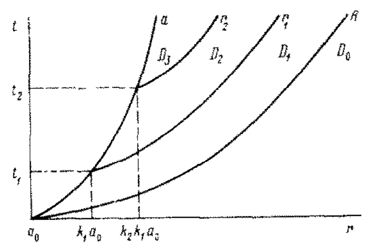

Fig.1

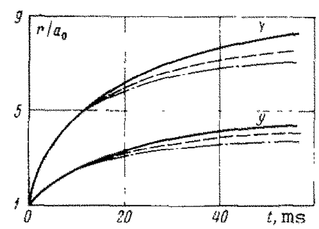

Fig.2

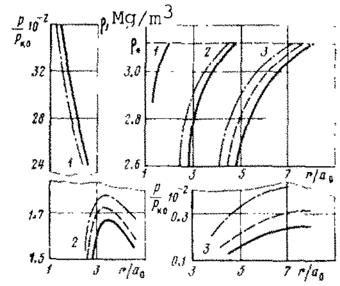

Fig.3

In each of the domains delineated the dilatancy equation (the last equation in (4)) can be written as

$$\partial(\rho^{\Lambda+1} r^{3\Lambda})/\partial t = 0$$

Integration yields the following expression for the density

$$\rho(r_0, t) = \begin{cases} \rho_0 & \text{in } D_0 \\ \rho_*(r_0/r(r_0,t))^{2-n_1} & \text{in } D_1 \\ \rho_1(k_1 r_0/r(r_0,t))^{2-n_2} & \text{in } D_2 \\ \rho_2 & \text{in } D_3 \end{cases} \tag{6}$$

$$n_i = (2-\Lambda_i)/(1+\Lambda_i),\quad i = 1, 2,\quad k_1 = (\rho_*/\rho_1)^{1/(2-n_1)}$$

Expression (6) helps us find the connection between the Lagrangian and the Euler coordinates of the points at which the dilatancy velocity changes:

$$r_1(t) = k_1 r_{01}(t),\ r_2(t) = k_1 k_2 r_{02}(t);\ k_2 = (\rho_1/\rho_2)^{1/(2-n_2)} \tag{7}$$

Integration of the equation of continuity (the second equation in (4)) enables us to express the Euler coordinate in terms of r_0 and $R(t)$:

$$r(r_0, t) = \begin{cases} r_0 & \text{in } D_0 \\ [(1-\varepsilon_1) r_0^{n_1+1} + \varepsilon_1 R^{n_1+1}(t)]^{1/(n_1+1)} & \text{in } D_1 \\ k_1[(1-\varepsilon_2) r_0^{n_2+1} + \varepsilon_2 c_1^{n_2+1} R^{n_2+1}(t)]^{1/(n_2+1)} & \text{in } D_2 \\ k_2 k_1 [(1-\varepsilon_3) r_0^3 + \varepsilon_3 c_1^3 c_2^3 R^3(t)]^{1/3} & \text{in } D_3 \end{cases} \tag{8}$$

$$\varepsilon_2 = 1 - \rho_0/(\rho_1 k_1^3),\quad \varepsilon_3 = 1 - \rho_0/(\rho_2 k_1^3 k_2^3)$$

$$c_i = [\varepsilon_i/(k_i^{n_i+1} + \varepsilon_i - 1)]^{1/(n_i+1)},\quad i = 1, 2$$

From formulas (7) and (8) it follows that

$$r_{01}(t) = c_1 R(t),\ r_{02}(t) = c_2 c_1 R(t) \tag{9}$$

Substitution into the left-hand side of equalities (9) the value of the cavity's initial radius a yields equations in t_1 and t_2. The functions $r_1(t)$ and $r_{01}(t)$ are defined for $t \geqslant t_1$, while the functions $r_2(t)$ and $r_{02}(t)$, for $t \geqslant t_2$. Differentiation of formulas (8) with respect to time leads to an expression for the medium's mass velocity

$$u(r_0, t) = \begin{cases} 0 & \text{in } D_0 \\ \varepsilon_1 R^{n_1}(t) R^{\cdot}(t)/r^{n_1}(r_0, t) & \text{in } D_1 \\ \varepsilon_2 k_1^{n_2+1} c_1^{n_2+1} R^{n_2}(t) R^{\cdot}(t)/r^{n_2}(r_0, t) & \text{in } D_2 \\ \varepsilon_3 k_1^3 k_2^3 c_1^3 c_2^3 R^2(t) R^{\cdot}(t)/r^2(r_0, t) & \text{in } D_3 \end{cases} \tag{10}$$

The equation of motion of the shock wave's front is obtained by integration of the first equality in (4) with respect to the Lagrangian radius from r_0 to $R(t)$ with due regard to formulas (3), (8) and (10). We introduce the following dimensionless variables:

$$\tau = t/\beta,\ x_0 = r_0/a_0,\ y(x_0, \tau) = r(a_0x_0, \beta\tau)/a_0,\ \pi(x_0, \tau) = p(a_0x_0, \beta\tau)/p_{k0},\ Y(\tau) = R(\beta\tau)/a_0$$

A unit of time is $\beta = a_0(\rho_0/p_{k0})^{1/2}$. In the dimensionless form the motion of the shock wave's front is described by the equation

$$A(x_0/Y)YY^{\cdot\cdot} + B(x_0/Y)Y^{\cdot 2} + G(\pi, x_0/Y) = 0 \tag{11}$$

The coefficients A, B, G in Eq.(11), depending on $x = x_0/Y$, take the values A_i, B_i, G_i, respectivly, in the domains D_i $(i = 1, 2, 3)$

$$A_1(x) = \varepsilon_1 \int_x^1 \xi^2 z_1^{\alpha-n_1-2}(\xi)\, d\xi$$

$$A_2(x) = A_1(c_1) + \varepsilon_2 k_1^{\alpha-1} c_1^{\alpha+2} \int_{x/c_1}^1 \xi^2 z_2^{\alpha-n_2-2}(\xi)\, d\xi$$

$$A_3(x) = A_2(c_1c_2) + \varepsilon_3 k_1^{\alpha-1} k_2^{\alpha-1} c_1^{\alpha+2} c_2^{\alpha+2} \int_{x/(c_1c_2)}^1 \xi^2 z_3^{\alpha-n_3-2}(\xi)\, d\xi$$

$$B_1(x) = \varepsilon_1 + n_1 A_1(x) - n_1\varepsilon_1^2 \int_x^1 \xi^2 z_1^{\alpha-2n_1-3}(\xi)\, d\xi$$

$$B_2(x) = B_1(c_1) + \varepsilon_2 k_1^{\alpha-1} c_1^{\alpha+2} n_2 \int_{x/c_1}^1 \xi^2 [z_2^{\alpha-n_2-2}(\xi) - \varepsilon_2 z_2^{\alpha-2n_2-3}(\xi)]\, d\xi$$

$$B_3(x) = B_2(c_1c_2) + 2\varepsilon_3 k_1^{\alpha-1} k_2^{\alpha-1} c_1^{\alpha+2} c_2^{\alpha+2} \times \int_{x/(c_1c_2)}^1 \xi^2 [z_3^{\alpha-n_3-2}(\xi) - \varepsilon_3 z_3^{\alpha-2n_3-3}(\xi)]\, d\xi$$

$$G_1(\pi, x) = q - z_1^{\alpha}(x)\pi_1(x_0, \tau),\ G_2(\pi, x) = q - k_1^{\alpha} c_1^{\alpha} z_2^{\alpha}(x/c_1)\pi_1(x_0, \tau)$$

$$G_3(\pi, x) = q - k_1^{\alpha} k_2^{\alpha} c_1^{\alpha} c_2^{\alpha} z_3^{\alpha}(x/(c_1c_2))\pi_1(x_0, \tau)$$

$$z_i(\xi) = [\varepsilon_i + (1 - \varepsilon_i)\xi^{n_i+1}]^{1/(n_i+1)}, \quad i = 1, 2, 3$$

$$q = (p_0 + k/(3m))/p_{k0},\ \pi_1(x_0, \tau) = \pi(x_0, \tau) + k/(3mp_{k0}),\ n_3 = 2$$

Setting $x_0 = 1$ and prescribing the pressure variation law at exploding cavity $\pi(1, \tau)$, Eq.(11) can be used for the numerical calculation of the dimensionless radius $Y(\tau)$ under the initial conditions

$$Y(0) = 1,\ Y^{\cdot}(0) = ((1 - p_0/p_{k0})/\varepsilon_1)^{1/2}$$

After $Y(\tau)$ is found, Eq.(11) yields, with the aid of (9), an explicit formula for the computation of the dimensionless pressure $\pi(x_0, \tau)$ over the whole zone over which the wave passed, $1 \leqslant x_0 \leqslant Y(\tau)$. The pressure at the cavity's boundary was taken in the form

$$\pi(1, \tau) = (a_0/r(a_0, t))^{3\gamma} \equiv y^{-3\gamma}(1, \tau)$$

which corresponds to an assumption on the adiabatic expansion of the cavern with a constant adiabat γ.

Figs.2 and 3 show the results of certain calculations with the following initial data: $m = 0.45$, $k = 34$ kPa, $\rho_0 = 2.5$ Mg/m^3, $\rho_1 = 2.85$ Mg/m^3, $\rho_2 = 2.6$ Mg/m^3, $\varepsilon_1 = 0.2$, $\Lambda_1 = 0.14$, $\Lambda_2 = 0.07$, $a_0 =$ 7 m, $p_{k0} = 6.2$ GPa, $\gamma = 1.5$. The unperturbed lithostatic pressure $p_0 = 10$; 17.5; 25 MPa, which roughly correspond to embedding depths of 400, 700 and 1000 m (the solid, dashed and dash-dotted curves, respectively). The dependence of the dimensionless front radius $Y = R/a_0$ and of the dimensionless cavity radius $y = a/a_0$ on time t (ms) is given in Fig.2. The mellowing of the rock being exploded at the expense of dilatancy and the dependence of the dimensionless pressure measured in units of $(p/p_{k0})\,10^{-2}$ on the dimensionless Euler radius r/a_0 are shown in Fig. 3 (curves *1*, *2*, *3* relate to the instants 0.8, 8.8 and 49 ms, respectively).

Relations (6)—(11) are generalized to the case of an arbitrary number of "steps" in (5) for the piecewise-constant approximation of $\Lambda(\rho)$.

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